EQUILIBRIUM RECONSTRUCTION FROM DISCRETE MAGNETIC MEASUREMENTS IN A TOKAMAK

Blaise Faugeras, Jacques Blum, Cedric Boulbe

EQUILIBRIUM RECONSTRUCTION FROM DISCRETE MAGNETIC MEASUREMENTS IN A TOKAMAK

Blaise Faugeras

(joint with Jacques Blum and Cedric Boulbe)

Laboratoire J.A Dieudonné, UMR 6621
Université de Nice Sophia-Antipolis, France
Blaise.Faugeras@unice.fr

ABSTRACT

We describe an algorithm for the reconstruction of the equilibrium in a Tokamak from discrete magnetic measurements. In order to solve this inverse problem we first use toroidal harmonics to compute Cauchy boundary conditions on a fixed closed contour. Then we use these Cauchy boundary conditions to solve a non-linear source identification problem.

1. INTRODUCTION

In this paper we are interested in the numerical reconstruction of the quasi-static equilibrium of a plasma in a Tokamak [1]. The state variable of interest in the modelization of such an equilibrium under the axisymmetric assumption is the poloidal flux $\psi(r,z)$ which is related to the poloidal magnetic field by the relation $B = \frac{1}{r}\nabla\psi^{\perp}$ in the cylindrical coordinate system (r,z).

A poloidal cross section of a Tokamak and the different domains and contours are shown on Fig. 1. The domain Ω_0 contains the poloidal field coils (PFcoils) domains Ω_{C_i} and the plasma domain Ω_p. It is assumed that Ω_0 does not include any ferromagnetic structure and thus the poloidal flux satisfies the elliptic PDE

$$-\Delta^*\psi = j \tag{1}$$

where the differential operator

$$\Delta^*. = \frac{\partial}{\partial r}\left(\frac{1}{\mu_0 r}\frac{\partial .}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\mu_0 r}\frac{\partial .}{\partial z}\right),$$

is linear and j is the toroidal component of the local current density.

In $\Omega_0 \setminus \{\Omega_p \bigcup \Omega_{C_i}\}$ the current is null ($j = 0$), in the PFcoils Ω_{C_i} it is supposed to be known ($j = I_i/S_i$, S_i is the surface of coil Ω_{C_i} and I_i the total intensity of the current) and in the plasma Ω_p it is unknown but takes the form

$$j = rp'(\psi) + \frac{1}{\mu_0 r}(ff')(\psi) \tag{2}$$

In Ω_p Eq. (1) is called the Grad-Shafranov equation and p' and ff' are unknown functions to be identified.

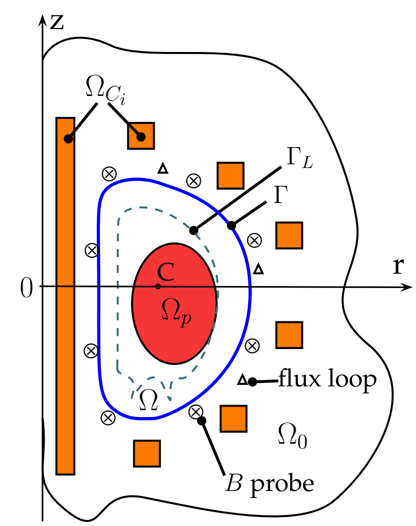

Figure 1: Poloidal cross section of a Tokamak.

The plasma domain is unknown, $\Omega_p = \Omega_p(\psi)$. It is a free boundary problem in which the plasma boundary is defined either by its contact with the limiter Γ_L (as in Fig. 1) or as a magnetic separatrix (hyperbolic line with an X-point).

In order to achieve the numerical reconstruction of the equilibrium the main inputs we have are magnetic measurements taken at several locations surrounding the vacuum vessel (see Fig. 1): B probes measure the local value of the poloidal magnetic field and flux loops measure the local value of the flux ψ.

The method we propose can be divided into two main steps described in the next section.

2. NUMERICAL METHOD

In a first step we solve Eq. (1) in $\Omega_0 \setminus \Omega_p$ using an analytic solution and a fit to the magnetic measurements. The goal is to transform the set of discrete measurements into Cauchy conditions for ψ on a fixed contour Γ. This latter defines a domain Ω of boundary Γ and containing the plasma (see Fig. 1). In a second step the value of ψ on Γ is used as a Dirichlet boundary condition to solve numerically Eq. (1) in Ω, whereas the remaining Neumann boundary condition on Γ is used simultaneously for the identification of the unknown functions p' and ff'.

2.1. Step 1 - Compute Cauchy conditions on Γ

Each of the PFcoils is modelized by a sum of filaments of current [3]. Using the analytic expression of the Green function of the operator Δ^* and the additivity property we can substract from ψ and from the magnetic measurements the effects of the PFcoils.

In the domain $\Omega_0 \setminus \Omega_p$ the resulting corrected flux then satisfies $-\Delta^*\psi = 0$. Using a system of toroidal coordinates with center C inside the plasma domain and a separation of variable technique, any solution of this equation can be shown to be equal to a series of toroidal harmonic functions T_n [5, 6]. Numerically these harmonics can be accurately evaluated [4] and the flux can be efficiently approximated using a truncated series, $\psi = \sum_{n=1}^{N} a_n T_n$. The coefficients a_n are computed by a least-square fit to the modified magnetic measurements. A regularization term can be added to the least-square cost function for example imposing some regularity on a circular contour surrounding the center of the toroidal coordinates system.

Finally one can evaluate $(\psi, \partial_n\psi)$ on the contour Γ and thus provide Cauchy conditions (g, h) to the resolution of the problem in the domain Ω.

2.2. Step 2 - Reconstruction in Ω

The Dirichlet boundary condition g is used to solve the boundary value problem:

$$\left\{ \begin{array}{rcl} -\Delta^*\psi & = & \lambda[\frac{r}{R_0}A(\bar{\psi}) + \frac{R_0}{r}B(\bar{\psi})]\chi_{\Omega_p(\psi)} \quad \text{in } \Omega \\ \psi & = & g \quad \text{on } \Gamma \end{array} \right. \tag{3}$$

where the unknown functions A and B are related to p' and ff', $\bar{\psi}$ is a normalized flux, and λ and R_0 are normalizing coefficients. The Neumann boundary condition is used to identify A and B by minimizing the cost function $J(A,B) = \int_\Gamma (\partial_n\psi - h)^2 ds + R$ where R is a Tikhonov regularization term.

An iterative strategy involving a finite element method for the resolution of the direct problem (3) and an optimisation procedure for the identification of the nonlinearity is proposed [2]. It is important to achieve this identification within a few ms so as to be able in the future to control the current profile in real time. The main ideas are: pre-computation of the inverse of the finite element stiffness matrix and of all the elements that are not modified by the non-linearities, Picard iterations for these non-linearities, reduction of the functions to be identified in small dimension basis and least-square resolution by normal equations.

3. CONCLUSION

The method presented here has led to the development of a software, EQUINOX, which enables to follow in real-time the quasi-static evolution of the plasma equilibrium in any Tokamak. It has already been validated on TORE SUPRA (the CEA-EURATOM Tokamak at Cadarache), JET (Joint European Torus) or ITER configurations.

4. REFERENCES

[1] J. Wesson, *Tokamaks*, Oxford University Press Inc., Third Edition, 2004.

[2] J. Blum, C. Boulbe and B. Faugeras, *Reconstruction of the equilibrium of the plasma in a Tokamak and identification of the current density profile in real time*, J. Comp. Phys., 2011.

[3] E. Durand, *Magnetostatique*, Masson et Cie, 1968.

[4] J. Segura, A. Gil, *Evaluation of toroidal harmonics*, Comp. Phys. Comm., 2000.

[5] N.N. Lebedev, *Special Functions and their Applications*, Dover Publications, 1972.

[6] Y. Fischer, *Approximation dans des classes de fonctions analytiques généralisées et résolution de problèmes inverses pour les tokamaks*, Thèse de Doctorat de l'Université de Nice-Sophia Antipolis, 2011